Theorem Let X_i denote the number of times that the outcome r_i occurs, $i = 1, 2, \ldots t$, in a series of n independent trials, where $p_i = P(r_i)$. Then the vector $(X_1, X_2, \ldots, X_t)$ has a multinomial distribution and

$$\begin{aligned} p_{X_1,X_2,\ldots,X_t}(k_1, k_2, \ldots, k_t) &= P(X_1 = k_1, X_2 = k_2, \ldots, X_t = k_t) \\ &= \frac{n!}{k_1! k_2! \cdots k_t!} p_1^{k_1} p_2^{k_2} \cdots p_t^{k_t}, \\ &\text{where } \; k_i = 0, 1, \ldots n; \;\; i = 1, 2, \ldots, t; \;\; \sum_{i=1}^{t} k_i = n \end{aligned}$$

Proof Any particular sequence of k_1 r_1's, k_2 r_2's, $\ldots$ and k_t r_t's has probability $p_1^{k_1} p_2^{k_2} \cdots p_t^{k_t}$. Moreover, the total number of outcome sequences that will generate the values $(k_1, k_2, ..., k_t)$ is the number of ways to permute n objects, k_1 of one type, k_2 of second type, $\ldots$, and k_t of a tth type. By a Theorem you learned in Probability Theory that number is $\frac{n!}{k_1! k_2! \cdots k_t!}$.